



Independent spanning trees on twisted cubes

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ABSTRACT

Multiple independent spanning trees have applications to fault tolerance and data broadcasting in distributed networks. There are two versions of the n independent spanning trees conjecture. The vertex (edge) conjecture is that any n -connected (n -edge-connected) graph has n vertex-independent spanning trees (edge-independent spanning trees) rooted at an arbitrary vertex. Note that the vertex conjecture implies the edge conjecture. The vertex and edge conjectures have been confirmed only for n -connected graphs with $n \leq 4$, and they are still open for arbitrary n -connected graph when $n \geq 5$. In this paper, we confirm the vertex conjecture (and hence also the edge conjecture) for the n -dimensional twisted cube TQ_n by providing an $O(N \log N)$ algorithm to construct n vertex-independent spanning trees rooted at any vertex, where N denotes the number of vertices in TQ_n . Moreover, all independent spanning trees rooted at an arbitrary vertex constructed by our construction method are isomorphic and the height of each tree is $n + 1$ for any integer $n \geq 2$.

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1. Introduction

Multiprocessor interconnect networks (in brief, interconnection networks or networks) play critical roles in parallel computing systems. An interconnection network can be represented by a graph, where a vertex is a processor and an edge is a communication link. In this paper, we use graphs and networks interchangeably.

Broadcasting, which is the process of sending a message from the source vertex to all other vertices in a network, has wide applications to the control of distributed systems and to parallel computing. Broadcasting can be accomplished by means of message dissemination in such a way that each vertex repeatedly receives and forwards messages. However, some of the vertices and/or links in networks may be faulty. We say that the broadcasting will be successful if all the healthy vertices in the network finally obtain the correct message from the source vertex within a certain limit of time.

There are two types of faults which are usually supposed in the study of broadcasting in networks. One is the crash type (i.e., a faulty vertex or link does not transmit any message) and the other is the Byzantine type (i.e., a faulty vertex or link may alter arbitrarily the messages that pass through it or even fabricate a false message). It has been shown that broadcasting using independent spanning trees is the most efficient and the most

reliable against the crash and Byzantine faults [3,2,23]. If G is an n -connected graph and there exist n independent spanning trees rooted at the source vertex on G , supposing that no vertices have priori information about faults in a network, fault tolerance can be achieved by sending n copies of the message along different channels (i.e., independent spanning trees rooted at the source vertex), so that a message corrupted at a place can spoil only one channel for each destination. If the source vertex is faultless, then there exists a broadcasting scheme that tolerates up to $n - 1$ faults of the crash type and up to $\lfloor (n - 1)/2 \rfloor$ faults of the Byzantine type even in the worst case [3].

Let $V(G)$ denote the vertex set and $E(G)$ denote the edge set of G . For two vertices $x, y \in V(G)$, let P and Q denote two paths from x to y in G . P and Q are *edge-disjoint* if $E(P) \cap E(Q) = \emptyset$. P and Q are *internally vertex-disjoint* if they are edge-disjoint and $V(P) \cap V(Q) = \{x, y\}$. A subgraph T of G is a *spanning tree* of G if T is a tree and $V(T) = V(G)$. Two spanning trees T and T' of G are *vertex-independent* (resp., *edge-independent*) if T and T' are rooted at the same vertex, say r , and for each vertex $v (\neq r)$ in G , the two paths from r to v , one path in each tree, are internally vertex-disjoint (resp., edge-disjoint). A set of spanning trees of G are *vertex-independent* (resp., *edge-independent*) if they are pairwise vertex-independent (resp., pairwise edge-independent).

The study of independent spanning trees started with Itai and Rodeh [23], where they proposed a multitree approach to reliability in distributed networks. They provided a linear time algorithm that, given any vertex r in a 2-connected graph G , finds two independent spanning trees of G rooted at r . Later, Cheriyan

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and Maheshwari [5] proved that for any vertex r in a 3-connected graph G , three independent spanning trees of G rooted at r can be found in $O(|V(G)||E(G)|)$ time.

Zehavi and Itai [39] also proved that every 3-connected graph contains three independent spanning trees rooted at any vertex, and they stated two versions of the n independent spanning trees conjecture:

- (1) Vertex Conjecture: Any n -connected graph has n vertex-independent spanning trees rooted at an arbitrary vertex.
- (2) Edge Conjecture: Any n -edge-connected graph has n edge-independent spanning trees rooted at an arbitrary vertex.

Zehavi and Itai raised the question: It would be interesting to show that either the vertex conjecture implies the edge conjecture, or vice versa. Khuller and Schieber [25] proved that the vertex conjecture implies the edge conjecture. The vertex and edge conjectures have been confirmed only for k -connected graphs with $k \leq 4$ in [5,6,23,24], and they are still open for arbitrary k -connected graphs when $k \geq 5$. However, by providing the construction schemes of independent spanning trees, the conjectures have been proved to hold for several restricted classes of graphs such as planar graphs [21–23,30,31], product graphs [32], hypercubes [33,34], locally twisted cubes [16,28,29], star graphs [14], and so forth. Note that the development of algorithms for constructing independent spanning trees tends toward pursuing two research goals, one is the design of efficient construction schemes [30,31] and the other is reducing the heights of independent spanning trees [15,23,38].

In general it is very hard to construct n independent spanning trees rooted at an arbitrary vertex of a given n -connected graph. In this paper we focus on the construction of independent spanning trees on twisted cubes.

The twisted cube [1] is an attractive variant of the hypercube. The diameter, wide diameter, and faulty diameter of n -dimensional twisted cube, denoted by TQ_n , are about half of those of the n -dimensional hypercube [1,4]. It was proved that TQ_n is $(n-2)$ -fault-tolerant Hamiltonian, $(n-3)$ -fault-tolerant Hamiltonian-connected [20]. In particular, a stronger result about Hamiltonian property can be seen in [36]. And paths [9,13], cycles [10], 2-dimensional meshes [26], multi-dimensional tori [27] can be embedded into twisted cube, respectively. According to [12], TQ_n is a bijective connection graph and thus has the same diagnosability as the n -dimensional hypercube under the t/k -diagnosis strategy based on the well-known PMC diagnostic model. Recently, in bijective connection networks, Fan et al. proposed efficient fault-tolerant routing algorithms with restricted faulty edge or node set [8,11], which are suitable for twisted cubes. There are also other variants of the hypercube, e.g., the crossed cube, locally twisted cube. The topological properties of hypercube and its variants have been discussed [7,17–19,37].

In [35], Yang proposed an $O(N \log N)$ algorithm to construct n edge-disjoint spanning trees on TQ_n , where N is the number of vertices in TQ_n . However, Yang did not solve the vertex conjecture for TQ_n .

In this paper, we provide an algorithm to generate n vertex-independent spanning trees rooted at an arbitrary vertex on TQ_n . Thus we confirm the vertex (also the edge) conjecture for TQ_n . Unless otherwise specified, in this paper, we use independent spanning trees to denote vertex-independent spanning trees.

The major contributions are as follows.

- (1) We improve the definition of twisted cubes, which extends the dimensions of twisted cubes to all the positive integers.
- (2) We prove that there are n independent spanning trees rooted at any vertex on TQ_n and present a recursive construction method.

- (3) We prove that all independent spanning tree rooted at an arbitrary vertex constructed by our construction method are isomorphic and the height of each tree is $n+1$ for any integer $n \geq 2$.
- (4) We give an $O(N \log N)$ algorithm to construct n independent spanning trees rooted at any vertex on TQ_n , where N denotes the number of vertices in TQ_n .

The rest of this paper is organized as follows: Section 2 provides the preliminaries. Section 3 gives the construction method of n independent spanning trees rooted at any vertex on TQ_n . Section 4 analyzes two properties of independent spanning trees constructed on TQ_n , i.e., the isomorphism and height. Section 5 gives an algorithm to construct n independent spanning trees rooted at any vertex on TQ_n and analyzes time complexity of the algorithm. In Section 6, we conclude this paper.

2. Preliminaries

Before presenting the construction method of independent spanning trees on twisted cubes, we first give some notations and related properties.

Given a simple graph G , if $V' \subseteq V(G)$, the subgraph of G induced by the vertex subset V' is denoted by $G[V']$. For a tree T rooted at a vertex r , the parent of a vertex x ($x \neq r$) in T is denoted by $\text{parent}(x, T)$, and we use $\text{ancestor}(r, x, T)$ to denote the set of vertices in the path from r to x in T . The height of T is the maximum distance of the paths from r to any other vertices in T , and we use $\text{Height}(T)$ to denote the height of T . Meanwhile, the degree of a vertex y in T (denoted by $\text{deg}(y, T)$) is defined as the number of its adjacent edges.

For any integer $n \geq 1$, a binary string u of length n is denoted by $u_{n-1}u_{n-2} \dots u_0$, where $u_i \in \{0, 1\}$ for any integer $i \in \{0, 1, \dots, n-1\}$. The complement of u_i is denoted by $\bar{u}_i = 1 - u_i$. Besides, u_0 is called the first bit of u and u_{n-1} is called the last bit of u . For any integer i with $0 \leq i \leq n-1$, we define $\theta(u, i) = u_i \oplus u_{i-1} \oplus \dots \oplus u_0$, where \oplus is the exclusive operation. Let $x = x_{m-1} \dots x_1x_0$ be a binary string of length m . We use xu to denote the binary string $x_{m-1}x_{m-2} \dots x_0u_{n-1}u_{n-2} \dots u_0$, whose length is $m+n$. Furthermore, let $U \subseteq \{0, 1\}^n$. That is, U is a set of some binary strings of length n , then we use xU to denote the set $\{xu|u \in U\}$. For any $(v, w) \in E(G)$, we use $x(v, w)$ to denote (xv, xw) , i.e., $x(v, w) = (xv, xw)$; and if $P \subseteq E(G)$, then we define $xP = \{x(v, w)|(v, w) \in P\}$.

A graph G_1 is isomorphic to another graph G_2 (denoted by $G_1 \cong G_2$) if and only if there exists a bijection $f: V(G_1) \rightarrow V(G_2)$ such that for any two vertices $u, v \in V(G_1)$, $(u, v) \in E(G_1)$ if and only if $(f(u), f(v)) \in E(G_2)$. We call f the isomorphic mapping function from G_1 to G_2 . For any $(u, v) \in E(G_1)$, we write $f(u, v) = (f(u), f(v))$. Furthermore, we write $f(V(G_1)) = V(G_2)$ and $f(E(G_1)) = E(G_2)$.

The n -dimensional twisted cube TQ_n is an n -regular graph of 2^n vertices. It can be recursively defined as below [13].

Definition 1. The 1-dimensional twisted cube TQ_1 is defined as the complete graph with two vertices labeled 0 and 1. For an odd integer $n \geq 3$, TQ_n consists of four subcubes TQ_{n-2}^{00} , TQ_{n-2}^{01} , TQ_{n-2}^{10} , and TQ_{n-2}^{11} , where TQ_{n-2}^{ab} is isomorphic to TQ_{n-2} and $V(TQ_{n-2}^{ab}) = \{abx|x \in V(TQ_{n-2})\}$ and $E(TQ_{n-2}^{ab}) = \{(abx, aby)|(x, y) \in E(TQ_{n-2})\}$ for $a, b \in \{0, 1\}$. $V(TQ_n) = \bigcup_{a,b \in \{0,1\}} V(TQ_{n-2}^{ab})$ and $E(TQ_n) = \bigcup_{a,b \in \{0,1\}} E(TQ_{n-2}^{ab}) \cup E'$, where for the vertices $u = u_{n-1}u_{n-2} \dots u_1u_0, v = v_{n-1}v_{n-2} \dots v_1v_0 \in V(TQ_n)$, $(u, v) \in E'$ if u and v satisfy one of the following three conditions:

- (1) $u = \bar{v}_{n-1}v_{n-2} \dots v_1v_0$;
- (2) $u = \bar{v}_{n-1}v_{n-2}v_{n-3} \dots v_1v_0$ and $\theta(u, n-3) = 0$;
- (3) $u = v_{n-1}\bar{v}_{n-2}v_{n-3} \dots v_1v_0$ and $\theta(u, n-3) = 1$.

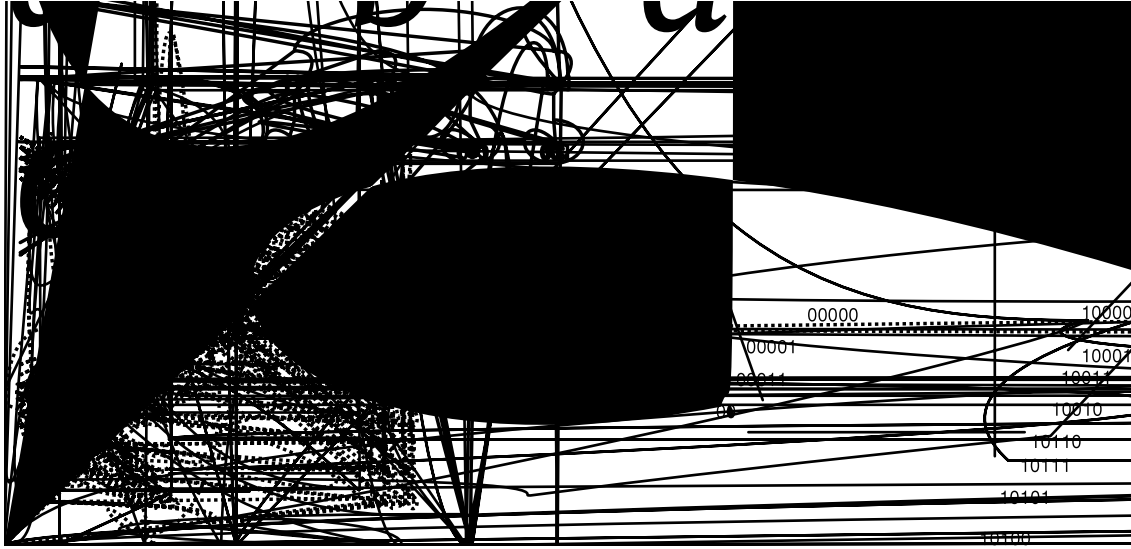


Fig. 1. (a) The 2-dimensional twisted cube TQ_2 . (b) The 3-dimensional twisted cube TQ_3 . (c) The 4-dimensional twisted cube TQ_4 . (d) The 5-dimensional twisted cube TQ_5 .

Fig. 1(b) and (d) demonstrate the 3-dimensional twisted cube TQ_3 and the 5-dimensional twisted cube TQ_5 .

Definition 1 is a recursive definition of TQ_n . For convenience of presentation, we present a non-recursive definition of TQ_n .

Definition 2. For any odd integer $n \geq 1$ and $u = u_{n-1}u_{n-2} \dots u_1u_0 \in V(TQ_n)$, $N_k(u)$ is defined as the k -dimensional adjacent vertex of u in TQ_n , where k is an integer with $1 \leq k \leq n$.

- (1) $N_1(u) = u_{n-1}u_{n-2} \dots u_1\bar{u}_0$;
- (2) if k is even, then $N_k(u) = u_{n-1}u_{n-2} \dots \bar{u}_k \dots u_1u_0$;
- (3) if k is odd and $\theta(u, k-3) = 0$, then $N_k(u) = u_{n-1}u_{n-2} \dots \bar{u}_{k-1}\bar{u}_{k-2} \dots u_1u_0$; otherwise, then $N_k(u) = u_{n-1}u_{n-2} \dots u_{k-1}\bar{u}_{k-2} \dots u_1u_0$.

Furthermore, if $V'' \subseteq V(TQ_n)$, then we define $N_k(V'') = \{N_k(u) | u \in V''\}$.

For example, let $u = 10000$ and $v = 10001$. Then $N_1(u) = 10001$, $N_2(u) = 10100$, $N_3(u) = 10110$, $N_4(u) = 00000$, $N_5(u) = 01000$; $N_1(v) = 10000$, $N_2(v) = 10101$, $N_3(v) = 10011$, $N_4(v) = 00001$, $N_5(v) = 11001$.

Definition 1 gives the definition of TQ_n for an odd integer $n \geq 1$. However, there is no definition of TQ_n for any even integer $n \geq 2$. In this paper, for convenience of discussion, we extend the recursive definition of TQ_n to any integer $n \geq 1$ as follows.

Definition 3. The 1-dimensional twisted cube TQ_1 is defined as the complete graph with two vertices labeled 0 and 1. For any integer $n \geq 1$,

- (1) if n is even, the n -dimensional twisted cube is divided into two types: 0-type n -dimensional twisted cube 0- TQ_n and 1-type n -dimensional twisted cube 1- TQ_n . For any integer $k \in \{0, 1\}$, $V(k-TQ_n) = \{ikx | i \in \{0, 1\}, x \in V(TQ_{n-1})\}$ and $E(k-TQ_n) = \bigcup_{i \in \{0, 1\}} ike(TQ_{n-1}) \cup E'$, where for any two vertices $u = u_{n-1}ku_{n-2} \dots u_1u_0$ and $v = v_{n-1}kv_{n-2} \dots v_1v_0 \in V(k-TQ_n)$, $(u, v) \in E'$ if $u = \bar{v}_{n-1}kv_{n-1} \dots v_1v_0$. For convenience of presentation, we write $jkTQ_{n-1}$ as TQ_{n-1}^j for $j \in \{0, 1\}$;
- (2) if n is odd, the definition of TQ_n is equivalent to that of TQ_n in **Definition 1**. For convenience of presentation, we write $k-TQ_{n-1}$ as TQ_{n-1}^k .

Whether n is odd or even, we always denote the n -dimensional twisted cube as TQ_n in this paper. Fig. 1(a) and (c) demonstrate the 2-dimensional twisted cube TQ_2 and the 4-dimensional twisted cube TQ_4 .

Based on **Definitions 1** and **3**, we can easily prove the following lemma.

Lemma 4. For any integers n and i with $n \geq 1$ and $i \in \{0, 1\}$, $TQ_n \cong TQ_n^i$.

Furthermore, based on the definitions of independent spanning trees and ancestor set, we have the following lemma.

Lemma 5. Let T and T' be two spanning trees rooted at vertex r in G . T and T' are independent if and only if for every vertex x in G , $x \neq r$, $\{r, x\} \subset \text{ancestor}(r, x, T) \cup \text{ancestor}(r, x, T')$ and $\text{ancestor}(r, x, T) \cap \text{ancestor}(r, x, T') = \{r, x\}$.

Notation 6. For any integers n and k with $n \geq 1$, $1 \leq k \leq n$ and $v \in V(TQ_n)$, supposing that there are n independent spanning trees T_1, T_2, \dots, T_n rooted at vertex v on TQ_n , if the root v takes $N_k(v)$ as its child in T_i , where i is an integer with $1 \leq i \leq n$, we use T_n^k to denote T_i .

Unless otherwise specified, we use $T_n^1, T_n^2, \dots, T_n^n$ to denote n independent spanning trees on TQ_n in this paper. And for convenience of explanation, we may represent a vertex in independent spanning trees by a decimal value. For example, four independent spanning trees rooted at 0 on TQ_4 shown in Fig. 2 are denoted by T_4^1, T_4^2, T_4^3 and T_4^4 from left to right, respectively.

3. Construction of independent spanning trees on TQ_n

In this section, we will discuss the construction of n independent spanning trees on TQ_n , where a recursive construction method which can generate n independent spanning trees rooted at any vertex on TQ_n from $n-1$ independent spanning trees on TQ_{n-1} is presented.

For any vertex $v \in V(TQ_{n-1})$, suppose that $n-1$ independent spanning trees $T_{n-1}^1, T_{n-1}^2, \dots, T_{n-1}^{n-1}$ rooted at v , where the child vertex of the root v in T_{n-1}^j ($j \in \{1, 2, \dots, n-1\}$) is $N_j(v)$, have been constructed on TQ_{n-1} . According to **Lemma 4**, we can obtain an isomorphic mapping function φ from TQ_{n-1} to TQ_{n-1}^0 .

Notation 7. For any integers n and i with $n \geq 2$ and $1 \leq i \leq n-1$, we use $T_{n-1}^{0,i}$ to denote $(\varphi(V(T_{n-1}^i)), \varphi(E(T_{n-1}^i)))$. That is, $T_{n-1}^{0,i} = (\varphi(V(T_{n-1}^i)), \varphi(E(T_{n-1}^i)))$.

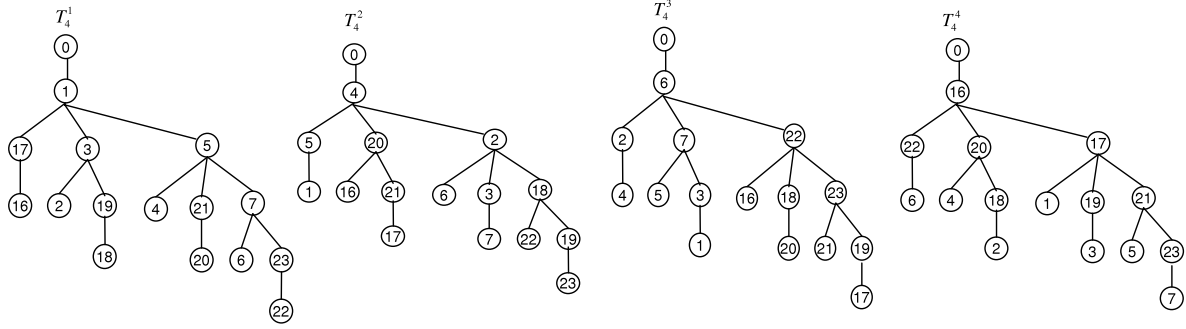


Fig. 2. Four independent spanning trees rooted at 0 on TQ_4 constructed by the proof of Theorem 17.

By Notation 7, we can easily prove the following lemma.

Lemma 8. $T_{n-1}^{0,1}, T_{n-1}^{0,2}, \dots,$ and $T_{n-1}^{0,n-1}$ are $n - 1$ independent spanning trees rooted at v on TQ_{n-1}^0 .

In the following, we will construct $n - 1$ independent spanning trees on TQ_{n-1}^0 by using $T_{n-1}^{0,1}, T_{n-1}^{0,2}, \dots,$ and $T_{n-1}^{0,n-1}$ on TQ_{n-1}^0 . We first give the following constructive rule.

Rule 9. For any integers n and k with $n \geq 2$ and $1 \leq k \leq n - 1$,

1. if n is even, by changing the last bit of each vertex in $T_{n-1}^{0,k}$ into its complement, we obtain one graph, denoted by $T_{n-1}^{1,k}$;
2. if n is odd, (i) if $\theta(v, n - 3) = 0$, by changing the $(n - 1)$ -th bit and $(n - 2)$ -th bit of each vertex in $T_{n-1}^{0,k}$ into their complements, we obtain one graph, denoted by $T_{n-1}^{1,k}$; (ii) if $\theta(v, n - 3) = 1$, by changing the $(n - 2)$ -th bit of each vertex in $T_{n-1}^{0,k}$ into its complement, we obtain one graph, denoted by $T_{n-1}^{1,k}$.

From Rule 9 and Definition 2, we can easily verify the following result.

Lemma 10. $T_{n-1}^{1,1}, T_{n-1}^{1,2}, \dots,$ and $T_{n-1}^{1,n-1}$ are $n - 1$ independent spanning trees rooted at $N_n(v)$ on TQ_{n-1}^1 . Furthermore, for any integer k with $1 \leq k \leq n - 1$, $T_{n-1}^{0,k} \cong T_{n-1}^{1,k}$.

Definition 11. For any vertex $u \in V(T_{n-1}^{0,k})$ ($1 \leq k \leq n - 1$), we define the corresponding vertex (CP(u)) of u in $T_{n-1}^{1,k}$ as follows:

- (1) for any even integer $n \geq 2$, let $u = u_{n-1}0u_{n-2} \dots u_1u_0 \in V(T_{n-1}^{0,k})$. Then $CP(u) = \overline{u_{n-1}}0u_{n-2} \dots u_1u_0 \in V(T_{n-1}^{0,k})$.
- (2) for any odd integer $n \geq 3$, let $u = u_{n-1}u_{n-2} \dots u_1u_0 \in V(T_{n-1}^{0,k})$. (i) if $\theta(v, n - 3) = 0$, then $CP(u) = \overline{u_{n-1}}u_{n-2} \dots u_1u_0$; (ii) if $\theta(v, n - 3) = 1$, then $CP(u) = u_{n-1}\overline{u_{n-2}} \dots u_1u_0$. If $V' \subset V(T_{n-1}^{0,k})$, we use $CP(V') = \{CP(u) | u \in V'\}$ to denote the corresponding vertex set of V' in $T_{n-1}^{1,k}$. Let G' be a subgraph of $T_{n-1}^{0,k}$, then $CP(G')$ is defined as the corresponding subgraph of G' in $T_{n-1}^{1,k}$, where $V(CP(G')) = \{CP(u) | u \in V(G')\}$ and $E(CP(G')) = \{(CP(u), CP(v)) | (u, v) \in E(G')\}$.

By Lemma 10 and Definition 11, the following lemma is obvious and its proof is omitted.

Lemma 12. $CP(G') \cong G'$.

From above discussion, we have constructed $n - 1$ independent spanning trees on TQ_{n-1}^1 by using $n - 1$ independent spanning trees on TQ_{n-1}^0 . In the following, we will give the definition of partition vertex. For any integer j with $1 \leq j \leq n - 1$, by connecting each partition vertex in $T_{n-1}^{0,j}$ with its n -dimensional adjacent vertex in $T_{n-1}^{1,j}$, we can construct the n independent spanning trees on TQ_n .

Definition 13. For any integers n and k with $n \geq 2$, $1 \leq k \leq n - 1$ and $v \in V(TQ_{n-1}^0)$, if $T_{n-1}^{0,k}$ is an independent spanning tree rooted

at v on TQ_{n-1}^0 and the child vertex of v is $N_k(v)$, then we recursively define a partition vertex set (ST_{n-1}^k) in $T_{n-1}^{0,k}$ as follows:

- (1) for $n = 2$, $ST_1^1 = \{N_1(v)\}$;
- (2) for any $n \geq 3$, $ST_{n-1}^i = ST_{n-2}^i \cup N_{n-1}(ST_{n-2}^i)$ for any integer i with $1 \leq i \leq n - 2$, and $ST_{n-1}^{n-1} = \{N_{n-1}(v)\}$.

We call a vertex in the partition vertex set ST_{n-1}^k a partition vertex in $T_{n-1}^{0,k}$.

For example, let the root $v = 0$. By Definition 2, $N_1(v) = 1$. According to Definition 13, $ST_1^1 = \{N_1(v)\} = \{1\}$. Similarly, $N_2(ST_1^1) = \{5\}$. Thus $ST_2^1 = ST_1^1 \cup N_2(ST_1^1) = \{1, 5\}$.

Based on Definitions 3, 11 and 13, we can give the following three associated lemmas.

Lemma 14. For any vertex $u \in V(T_{n-1}^{0,k})$, let $w = N_{n-1}(u)$. Then

- (1) for any even integer $n \geq 2$, $CP(u) = N_n(u)$.
- (2) for any odd integer $n \geq 3$, if $\theta(v, n - 3) = \theta(u, n - 3)$, then $CP(u) = N_n(u)$; otherwise, $CP(w) = N_n(u)$ and $CP(u) = N_n(w)$.

Proof. (1) For any even integer $n \geq 2$, let $u = u_{n-1}0u_{n-2} \dots u_1u_0 \in V(T_{n-1}^{0,k})$. By Definition 11, $CP(u) = \overline{u_{n-1}}0u_{n-2} \dots u_1u_0$; and by Definition 2, $N_n(u) = \overline{u_{n-1}}0u_{n-2} \dots u_1u_0$. Thus $CP(u) = N_n(u)$.

(2) For any odd integer $n \geq 3$, let $u = u_{n-1}u_{n-2} \dots u_1u_0 \in V(T_{n-1}^{0,k})$. We deal with the following cases.

Case 1. $\theta(v, n - 3) = 0$. By Definition 11, $CP(u) = \overline{u_{n-1}}u_{n-2} \dots u_1u_0$. We have the following sub-cases.

Case 1.1. $\theta(u, n - 3) = 0$. By Definition 2, $N_n(u) = \overline{u_{n-1}}u_{n-2} \dots u_1u_0$. Therefore, $CP(u) = N_n(u)$.

Case 1.2. $\theta(u, n - 3) = 1$. By Definition 2, $N_n(u) = u_{n-1}\overline{u_{n-2}} \dots u_1u_0$. Since $w = N_{n-1}(u) = \overline{u_{n-1}}u_{n-2} \dots u_1u_0$, then by Definitions 11 and 2, $CP(w) = u_{n-1}\overline{u_{n-2}} \dots u_1u_0$ and $N_n(w) = \overline{u_{n-1}}u_{n-2} \dots u_1u_0$. Therefore, $CP(w) = N_n(u)$ and $CP(u) = N_n(w)$.

Case 2. $\theta(v, n - 3) = 1$. By Definition 11, $CP(u) = u_{n-1}\overline{u_{n-2}} \dots u_1u_0$. We have the following sub-cases.

Case 2.1. $\theta(u, n - 3) = 1$. By Definition 2, $N_n(u) = u_{n-1}\overline{u_{n-2}} \dots u_1u_0$. Therefore, $CP(u) = N_n(u)$.

Case 2.2. $\theta(u, n - 3) = 0$. By Definition 2, $N_n(u) = \overline{u_{n-1}}u_{n-2} \dots u_1u_0$. Since $w = N_{n-1}(u) = \overline{u_{n-1}}u_{n-2} \dots u_1u_0$, by Definitions 2 and 11, $CP(w) = \overline{u_{n-1}}u_{n-2} \dots u_1u_0$ and $N_n(w) = u_{n-1}\overline{u_{n-2}} \dots u_1u_0$. Therefore, $CP(w) = N_n(u)$ and $CP(u) = N_n(w)$.

In summary, the lemma holds. \square

According to Definition 13, for any integers n and i with $n \geq 3$ and $1 \leq i \leq n - 2$, $ST_{n-1}^i = ST_{n-2}^i \cup N_{n-1}(ST_{n-2}^i)$. Thus for any $u \in ST_{n-2}^i$, there exists a vertex $w \in N_{n-1}(ST_{n-2}^i)$ such that $w = N_{n-1}(u)$. Moreover, by Lemma 14, $\{CP(u), CP(w)\} = \{N_n(u), N_n(w)\}$. Therefore, we can easily verify the following lemma.

Lemma 15. $N_n(ST_{n-1}^i) = CP(ST_{n-1}^i)$.

Lemma 16. For any integers n, i and j with $n \geq 2, 1 \leq i, j \leq n - 1, i \neq j$ and $v \in V(TQ_{n-1}^0)$, ST_{n-1}^i is a partition vertex set in $T_{n-1}^{0,i}$ rooted at v . Then (1) $\bigcup_{i=1}^{n-1} ST_{n-1}^i = V(T_{n-1}^{0,i}) - \{v\}$; (2) $ST_{n-1}^i \cap ST_{n-1}^j = \emptyset$.

Proof. We will prove the lemma by induction on n .

The lemma clearly holds for $n = 2$. Supposing that the lemma holds for $n = k - 1$ ($k \geq 3$), we consider $n = k$.

By the induction hypothesis, $\bigcup_{i=1}^{k-2} ST_{k-2}^i = V(T_{k-2}^{0,i}) - \{v\}$ and $ST_{k-2}^i \cap ST_{k-2}^j = \emptyset$. By Lemma 10, $T_{k-2}^{0,i} \cong T_{k-2}^{1,i}$, thus $\bigcup_{i=1}^{k-2} CP(ST_{k-2}^i) = V(T_{k-2}^{1,i}) - \{N_{k-1}(v)\}$ and $CP(ST_{k-2}^i) \cap CP(ST_{k-2}^j) = \emptyset$. Meanwhile, by Lemma 15, $N_{k-1}(ST_{k-2}^i) = CP(ST_{k-2}^i)$. Thus $\bigcup_{i=1}^{k-2} N_{k-1}(ST_{k-2}^i) = V(T_{k-2}^{1,i}) - \{N_{k-1}(v)\}$ and $N_{k-1}(ST_{k-2}^i) \cap N_{k-1}(ST_{k-2}^j) = \emptyset$. Furthermore, $(\bigcup_{i=1}^{k-2} ST_{k-2}^i) \cup (\bigcup_{i=1}^{k-2} N_{k-1}(ST_{k-2}^i)) = (V(T_{k-2}^{0,i}) - \{v\}) \cup (V(T_{k-2}^{1,i}) - \{N_{k-1}(v)\}) = V(T_{k-1}^{0,i}) - \{v, N_{k-1}(v)\}$. By Definition 13, $ST_{k-1}^i = ST_{k-2}^i \cup N_{k-1}(ST_{k-2}^i)$ and $ST_{k-1}^{k-1} = \{N_{k-1}(v)\}$. As a result, $\bigcup_{i=1}^{k-1} ST_{k-1}^i = (\bigcup_{i=1}^{k-2} ST_{k-2}^i) \cup (\bigcup_{i=1}^{k-2} N_{k-1}(ST_{k-2}^i)) \cup ST_{k-1}^{k-1} = V(T_{k-1}^{0,i}) - \{v\}$. Since $ST_{k-2}^i \cap N_{k-1}(ST_{k-2}^j) = \emptyset$, $ST_{k-1}^i \cap ST_{k-1}^j = (ST_{k-2}^i \cup N_{k-1}(ST_{k-2}^i)) \cap (ST_{k-2}^j \cup N_{k-1}(ST_{k-2}^j)) = (ST_{k-2}^i \cap ST_{k-2}^j) \cup (N_{k-1}(ST_{k-2}^i) \cap N_{k-1}(ST_{k-2}^j)) = \emptyset$. In summary, the lemma holds for $n = k$. \square

Theorem 17. For any integer $n \geq 1$ and $v \in V(TQ_n)$, there exist n independent spanning trees rooted at v on TQ_n .

Proof. We will prove the theorem by induction on n .

(1) For $n = 1$, there is one independent spanning tree T_1^1 rooted at v on TQ_1 , where $V(T_1^1) = \{v, N_1(v)\}$ and $E(T_1^1) = \{(v, N_1(v))\}$. Therefore, the theorem clearly holds for $n = 1$.

(2) Supposing that the theorem holds for $n = k - 1$ ($k \geq 2$), we consider $n = k$.

In the following, we first construct T_k^1, T_k^2, \dots , and T_k^k rooted at v on TQ_k , then we prove T_k^1, T_k^2, \dots , and T_k^k are independent spanning trees on TQ_k .

By the induction hypothesis, there exist $k - 1$ independent spanning trees $T_{k-1}^1, T_{k-1}^2, \dots$, and T_{k-1}^{k-1} rooted at v on TQ_{k-1} . By Lemmas 4 and 8, we can obtain $k - 1$ independent spanning trees $T_{k-1}^{0,1}, T_{k-1}^{0,2}, \dots$, and $T_{k-1}^{0,k-1}$ rooted at v on TQ_{k-1}^0 . According to Rule 9, we can construct $k - 1$ independent spanning trees $T_{k-1}^{1,1}, T_{k-1}^{1,2}, \dots$, and $T_{k-1}^{1,k-1}$ rooted at $N_k(v)$ on TQ_{k-1}^1 . Furthermore, by Definition 13, $ST_{k-1}^i = \{N_1(v)\}$ and for any integer i with $1 \leq i \leq k - 1$, $ST_{k-1}^i = ST_{k-1}^i \cup N_k(ST_{k-1}^i)$ and $ST_{k-1}^k = N_k(v)$.

For $v \in V(TQ_k)$, we have the following cases.

Case 1. $v \in V(TQ_{k-1}^0)$. The construction of T_k^1, T_k^2, \dots , and T_k^k can be described as follows.

Step 1: For any vertex $u \in V(T_{k-1}^{0,i})$, if $u \in ST_{k-1}^i$, then parent $(N_k(u), T_k^i) = u$. Let $V(T_k^i) = V(T_{k-1}^{0,i}) \cup V(T_{k-1}^{1,i})$ and $E(T_k^i) = (E(T_{k-1}^{0,i}) \cup E(T_{k-1}^{1,i}) \cup E') - E''$, where $E' = \{(u, N_k(u)) | u \in ST_{k-1}^i\}$ and $E'' = E(T_{k-1}^{1,i}[N_k(ST_{k-1}^i)])$.

Step 2: We have the following sub-steps to construct T_k^k .

Step 2.1: Let $N_k(v)$ be the single child of the root v .

Step 2.2: For any vertex $u \in V(T_{k-1}^{1,i})$, if $u \in N_k(ST_{k-1}^i)$, then parent $(u, T_k^k) = \text{parent}(u, T_{k-1}^{1,i})$.

Step 2.3: For any vertex $w \in V(T_{k-1}^{0,i})$, if $w \in ST_{k-1}^i$, then parent $(w, T_k^k) = N_k(w)$.

Based on above steps, for $v \in V(TQ_{k-1}^0)$, T_k^1, T_k^2, \dots , and T_k^k rooted at v on TQ_k have been constructed.

Case 2. $v \in V(TQ_{k-1}^1)$. The construction of T_k^1, T_k^2, \dots , and T_k^k can be described as follows.

Step 1: For any vertex $u \in V(T_{k-1}^{0,i})$, if $u \in ST_{k-1}^i$, then parent $(u, T_k^i) = N_k(u)$. Let $V(T_k^i) = V(T_{k-1}^{0,i}) \cup V(T_{k-1}^{1,i})$ and $E(T_k^i) = (E(T_{k-1}^{0,i}) \cup E(T_{k-1}^{1,i}) \cup E') - E''$, where $E' = \{(u, N_k(u)) | u \in ST_{k-1}^i\}$ and $E'' = E(T_{k-1}^{0,i}[ST_{k-1}^i])$.

Step 2: We have the following sub-steps to construct T_k^k .

Step 2.1: Let $N_k(v)$ be the single child of the root v .

Step 2.2: For any vertex $u \in V(T_{k-1}^{0,i})$, if $u \in ST_{k-1}^i$, then parent

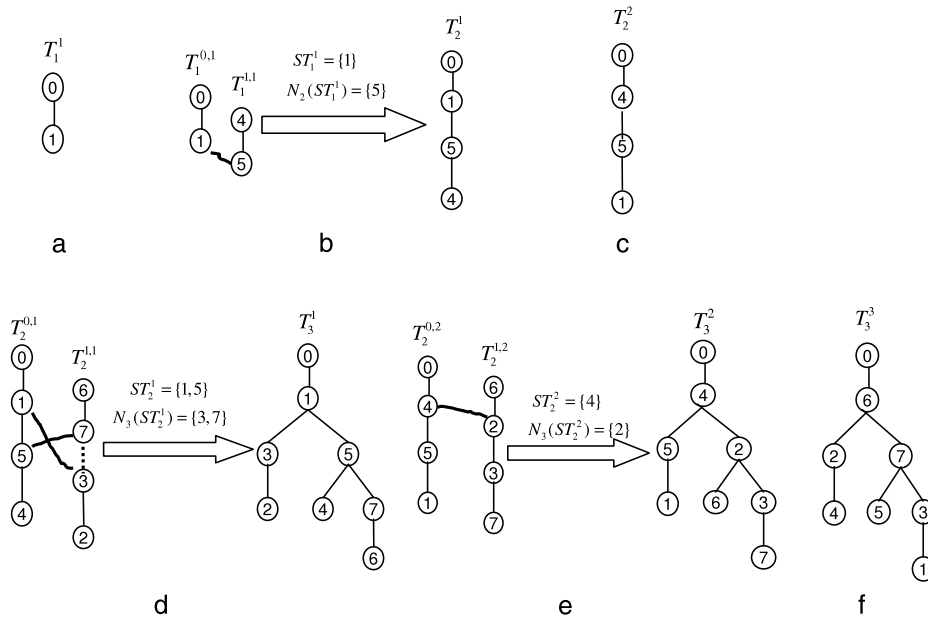


Fig. 3. The construction of three independent spanning trees rooted at 0 on TQ_3 according to the proof of Theorem 17.

Case 2. $u \in V(TQ_{k-1}^1)$. We deal with the following sub-cases.

Case 2.1. u is the single child of root v in T_k^k . Based on above construction method, $\text{ancestor}(v, u, T_k^k) = \{v, u\}$ and $\{v, u\} \subset \text{ancestor}(v, u, T_k^p)$. Thus, $\{v, u\} \subset \text{ancestor}(v, u, T_k^p) \cup \text{ancestor}(v, u, T_k^k)$ and $\text{ancestor}(v, u, T_k^p) \cap \text{ancestor}(v, u, T_k^k) = \{v, u\}$.

Case 2.2. u is not the single child of root v in T_k^k . By above construction method and Lemma 16, there exists an integer m with $1 \leq m \leq k - 1$ such that $u \in N_k(ST_{k-1}^m)$. Based on above construction method, $\text{parent}(u, T_k^m) \in ST_{k-1}^m$. Thus $(\text{ancestor}(v, u, T_k^m) - \{u\}) \subset V(TQ_{k-1}^0)$. Meanwhile, based on the above constructive method, $\text{parent}(u, T_k^k) \in V(TQ_{k-1}^1)$. Thus, $\{v, u\} \subset \text{ancestor}(v, u, T_k^k)$ and $(\text{ancestor}(v, u, T_k^k) - \{v\}) \subset V(TQ_{k-1}^1)$. As a result, $\{v, u\} \subset \text{ancestor}(v, u, T_k^m) \cup \text{ancestor}(v, u, T_k^k)$ and $\text{ancestor}(v, u, T_k^m) \cap \text{ancestor}(v, u, T_k^k) = \{v, u\}$.

Based on Case 1, Case 2 and Lemma 5, we have proven that T_k^p ($1 \leq p \leq k - 1$) and T_k^k are independent.

Therefore, $T_k^1, T_k^2, \dots,$ and T_k^k are k independent spanning trees rooted at any vertex v on TQ_k .

Thus the theorem holds. \square

The construction of three independent spanning trees rooted at 0 on TQ_3 is shown in Fig. 3, where E'' is shown by dotted lines.

4. Two properties of independent spanning trees

In fact, Theorem 17 gives a construction method of n independent spanning trees rooted at any vertex v on TQ_n . In Section 5, we will present an algorithm and analyze its time complexity. In this section, we discuss two properties of independent spanning trees constructed by the proof of Theorem 17, that is, isomorphism and the height of these trees. By the proof of Theorem 17, for $v \in V(TQ_{n-1}^0)$ or $v \in V(TQ_{n-1}^1)$, the construction methods are similar. Without loss of generality, in the following, we simply consider $v \in V(TQ_{n-1}^0)$.

4.1. Isomorphism of independent spanning trees

In order to prove that a set of independent spanning trees constructed by the proof of Theorem 17 are isomorphic. We first give the following lemma.

Lemma 18. A set of independent spanning trees constructed on TQ_n by the proof of Theorem 17 satisfy the following properties: for any integers n and k with $n \geq 3, 1 \leq k \leq n - 1$ and two vertices $u, v \in V(TQ_n), T_n^k$ is an independent spanning tree rooted at vertex v on TQ_n , where the child of v is $N_k(v)$.

- (1) If u is the single child of root v in T_n^n , then $\text{deg}(u, T_n^n) = n$;
- (2) if $\text{deg}(u, T_n^k) \geq k + 1$, then $u \in ST_n^k$;
- (3) if $u \in ST_n^k$, then $\text{deg}(u, T_n^k) \geq k + 1$; and if $u \in N_n(ST_{n-1}^k)$, then $\text{deg}(u, T_n^k) = k + 1$;
- (4) if $\text{deg}(u, T_n^k) = k + 1$, then $u = N_n(p)$, where $p = \text{parent}(u, T_n^k)$.

Proof. (1) By the proof of Theorem 17, in T_n^n the single child of root v is $N_n(v)$. Since $N_n(v)$ is the root of $T_{n-1}^{1,k}$, by the proof of Theorem 17, $N_n(v)$ has $n - 1$ child vertices in T_n^n . Therefore, $\text{deg}(u, T_n^n) = (n - 1) + 1 = n$.

In the following, we will prove (2), (3) and (4) by induction on n . It can be easily verified that the lemma holds for $n = 3$. Supposing that the lemma holds for $n = m - 1$ ($m \geq 4$), we consider $n = m$.

(2) For $1 \leq k \leq m - 1$, we have the following cases.

Case 1. $1 \leq k \leq m - 2$. By the induction hypothesis, in $T_{m-1}^{0,k}$ all the vertices whose degree is greater than or equal to $k + 1$ are partition vertices. Meanwhile, by Lemma 10, $T_{m-1}^{0,k} \cong T_{m-1}^{1,k}$. Furthermore, according to Lemma 15, $N_m(ST_{m-1}^k) = \text{CP}(ST_{m-1}^k)$. Thus each vertex whose degree in $T_{m-1}^{1,k}$ is greater than or equal to $k + 1$ is an m -dimensional adjacent vertex of a partition vertex in $T_{m-1}^{0,k}$. According to the proof of Theorem 17, T_m^k is constructed by connecting each partition vertex in $T_{m-1}^{0,k}$ with its m -dimensional adjacent vertex in $T_{m-1}^{1,k}$, and $V(T_m^k) = V(T_{m-1}^{0,k}) \cup V(T_{m-1}^{1,k})$. Therefore, the degree of the vertices whose degree is less than $k + 1$ in $T_{m-1}^{0,k}$ and $T_{m-1}^{1,k}$ remains unchanged in T_m^k . As a result, for any $u \in V(T_m^k)$, if $\text{deg}(u, T_m^k) \geq k + 1$, then $u \in ST_{m-1}^k$ or $u \in N_m(ST_{m-1}^k)$. Furthermore, since $ST_m^k = ST_{m-1}^k \cup N_m(ST_{m-1}^k)$, u is a partition vertex in T_m^k . That is, $u \in ST_m^k$.

Case 2. $k = m - 1$. According to the proof of Theorem 17, in $T_{m-1}^{0,m-1}, ST_{m-1}^{m-1} = N_{m-1}(v)$ and $N_{m-1}(v)$ is the single child of root v . By (1), $\text{deg}(N_{m-1}(v), T_{m-1}^{0,m-1}) = m - 1$. Meanwhile, by Lemma 10, $T_{m-1}^{0,k} \cong T_{m-1}^{1,k}$. Thus $\text{deg}(\text{CP}(N_{m-1}(v)), T_{m-1}^{1,m-1}) = m - 1$. According to Lemma 14, $N_m(N_{m-1}(v)) = \text{CP}(N_{m-1}(v))$. Thus

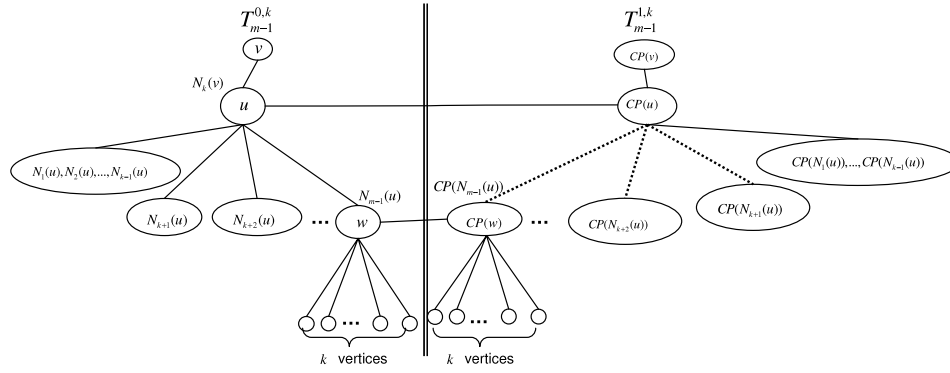


Fig. 4. The computation of $\deg(\text{CP}(u), T_m^k)$ where $u \in \text{ST}_{m-2}^k$, u is the single child of root v and parent $(\text{CP}(u), T_m^k) = u$.

$\deg(N_m(N_{m-1}(v)), T_{m-1}^{1,m-1}) = m - 1$. Furthermore, by the proof of [Theorem 17](#), $\deg(N_{m-1}(v), T_{m-1}^{m-1}) = \deg(N_m(N_{m-1}(v)), T_{m-1}^{m-1}) = m$. Thus in T_{m-1}^{m-1} , the degree of vertices except $N_{m-1}(v)$ and $N_m(N_{m-1}(v))$ is less than m , i.e., $k + 1$. By the proof of [Theorem 17](#), the degree of the vertices whose degree is less than $k + 1$ in $T_{m-1}^{0,m-1}$ and $T_{m-1}^{1,m-1}$ remains unchanged in T_m^{m-1} . Thus, for any $u \in V(T_{m-1}^{m-1})$, if $\deg(u, T_{m-1}^{m-1}) = k + 1$, then $u \in \text{ST}_{m-1}^{m-1}$ or $u \in N_m(\text{ST}_{m-1}^{m-1})$. Furthermore, since $\text{ST}_m^k = \text{ST}_{m-1}^k \cup N_m(\text{ST}_{m-1}^k)$, u is a partition vertex in T_m^k . That is, $u \in \text{ST}_m^k$.

From Cases 1 and 2, for $1 \leq k \leq m - 1$, if $\deg(u, T_m^k) \geq k + 1$, then $u \in \text{ST}_m^k$.

(3) Since $\text{ST}_m^k = \text{ST}_{m-1}^k \cup N_m(\text{ST}_{m-1}^k)$, for any $s \in \text{ST}_m^k$, we further have the following cases:

Case 1. $s \in \text{ST}_{m-1}^k$. By the proof of [Theorem 17](#), parent $(N_m(s), T_m^k) = s$. Thus, $\deg(s, T_m^k) = \deg(s, T_{m-1}^{0,k}) + 1$. By the induction hypothesis, for $1 \leq k \leq m - 2$, $\deg(s, T_{m-1}^{0,k}) \geq k + 1$. Furthermore, by the proof of [Theorem 17](#), $\text{ST}_{m-1}^{m-1} = N_{m-1}(v)$. By (1), $\deg(s, T_{m-1}^{m-1}) = m - 1$. As a result, for $1 \leq k \leq m - 1$, $\deg(s, T_m^k) \geq k + 1$.

Case 2. $s \in N_m(\text{ST}_{m-1}^k)$. Let $u = \text{parent}(s, T_m^k)$. By the proof of [Theorem 17](#), thus $u \in \text{ST}_{m-1}^k$ and $s = N_m(u)$. Since $\text{ST}_{m-1}^k = \text{ST}_{m-2}^k \cup N_{m-1}(\text{ST}_{m-2}^k)$, we have the following cases.

Case 2.1. $u \in \text{ST}_{m-2}^k$. Let $w = N_{m-1}(u)$. Then $w \in N_{m-1}(\text{ST}_{m-2}^k)$. By [Definition 13](#), $\text{ST}_{m-1}^k = \text{ST}_{m-2}^k \cup N_{m-1}(\text{ST}_{m-2}^k)$. Thus $w \in \text{ST}_{m-1}^k$. By the induction hypothesis, $\deg(w, T_{m-1}^k) = k + 1$. By [Lemma 14](#), $N_m(u) = \text{CP}(u)$ or $N_m(u) = \text{CP}(w)$, i.e., $s = \text{CP}(u)$ or $s = \text{CP}(w)$. In the following, we will prove $\deg(s, T_m^k) = k + 1$. We deal with the following cases.

Case 2.1.1. $s = \text{CP}(u)$. We further have the following sub-cases.

Case 2.1.1.1. u is the single child of root v in $T_{m-1}^{0,k}$. By [Notation 6](#), $u = N_k(v)$. Meanwhile, by the proof of [Theorem 17](#), $N_k(v)$ is the single child of root v in T_k^k . Furthermore, by (1), $\deg(N_k(v), T_k^k) = k$. According to the proof of [Theorem 17](#), among the child vertices of u , there are $m - k - 1$ partition vertices in $T_{m-1}^{0,k}$, which are $N_{k+1}(u), N_{k+2}(u), \dots$, and $N_{m-1}(u)$. Therefore, $\deg(u, T_{m-1}^{0,k}) = k + (m - k - 1) = m - 1$. And by [Lemma 10](#), $T_{m-1}^{0,k} \cong T_{m-1}^{1,k}$. Therefore, $\deg(\text{CP}(u), T_{m-1}^{1,k}) = m - 1$. Then by the proof of [Theorem 17](#), in $T_{m-1}^{1,k}$ the edge set $\{(\text{CP}(u), \text{CP}(N_{k+1}(u))), (\text{CP}(u), \text{CP}(N_{k+2}(u))), \dots, (\text{CP}(u), \text{CP}(N_{m-1}(u)))\} \subset E''$ (E'' is defined in the proof in [Theorem 17](#) and shown by dotted lines in [Fig. 4](#)). Therefore, $\deg(s, T_m^k) = \deg(\text{CP}(u), T_m^k) = \deg(\text{CP}(u), T_{m-1}^{1,k}) - (m - k - 1) + 1 = k + 1$ (see [Fig. 4](#)).

Case 2.1.1.2. u is not the single child of the root v in $T_{m-1}^{0,k}$. Then there exists an integer l with $1 \leq l \leq m - 1$ such that $u \in N_l(\text{ST}_{l-1}^k)$. By the proof of [Theorem 17](#), among the child vertices of u are $m - l - 1$ partition vertices, which are $N_{l+1}(u), N_{l+2}(u), \dots$,

and $N_{m-1}(u)$. By the induction hypothesis, $\deg(w, T_{m-1}^{0,k}) = k + 1$. Thus, $\deg(u, T_{m-1}^{0,k}) = k + 1 + (m - 1) - (l + 1) + 1 = k + m - l \geq k + 1$. Meanwhile, by [Lemma 10](#), $T_{m-1}^{0,k} \cong T_{m-1}^{1,k}$. Therefore, $\deg(\text{CP}(u), T_{m-1}^{1,k}) = k + m - l$. Since there are $m - l - 1$ partition vertices among the $k + m - l - 1$ child vertices of u , then $\deg(s, T_m^k) = \deg(\text{CP}(u), T_m^k) = \deg(\text{CP}(u), T_{m-1}^{1,k}) - (m - l - 1) - 1 + 1 = k + 1$ (see [Fig. 5](#)).

Case 2.1.2. $s = \text{CP}(w)$. By the induction hypothesis, $\deg(w, T_{m-1}^{0,k}) = k + 1$. By the proof of [Theorem 17](#), it is impossible for the child vertices of w to be partition vertices in $T_{m-1}^{0,k}$. Therefore, $\deg(s, T_m^k) = \deg(\text{CP}(w), T_m^k) = \deg(\text{CP}(w), T_{m-1}^{1,k}) - 1 + 1 = k + 1$.

From Case 2.1.1 and Case 2.1.2, we have the result: $\deg(s, T_m^k) = \deg(\text{CP}(u), T_m^k) = k + 1$, where $u \in \text{ST}_{m-2}^k$.

Case 2.2. $u \in N_{m-1}(\text{ST}_{m-2}^k)$. Let $p = \text{parent}(u, T_{m-1}^k)$. Then by the proof of [Theorem 17](#), $p \in \text{ST}_{m-2}^k$ and $u = N_{m-1}(p)$. By the induction hypothesis, $\deg(u, T_{m-1}^k) = k + 1$. By [Lemma 14](#), $N_m(u) = \text{CP}(u)$ or $N_m(u) = \text{CP}(p)$, i.e., $s = \text{CP}(u)$ or $s = \text{CP}(p)$. In the following, we will prove $\deg(s, T_m^k) = k + 1$. We deal with the following cases.

Case 2.2.1. $s = \text{CP}(u)$. Similar to the computation of $\text{CP}(w)$ in Case 2.1.2, thus $\deg(s, T_m^k) = k + 1$.

Case 2.2.2. $s = \text{CP}(p)$. Similar to the computation of $\text{CP}(u)$ in Case 2.1.1, thus $\deg(s, T_m^k) = k + 1$.

From Cases 1 and 2, we have proven that if $u \in \text{ST}_m^k$, then $\deg(u, T_m^k) \geq k + 1$; and if $u \in N_m(\text{ST}_{m-1}^k)$, then $\deg(u, T_m^k) = k + 1$.

(4) Let u denote an arbitrary vertex in T_m^k whose degree is greater than or equal to $k + 1$. By (2), thus $u \in \text{ST}_m^k$. Since $\text{ST}_m^k = \text{ST}_{m-1}^k \cup N_m(\text{ST}_{m-1}^k)$, if $u \in \text{ST}_{m-1}^k$, by the induction hypothesis, $\deg(u, T_{m-1}^{0,k}) \geq k + 1$. Furthermore, by the proof of [Theorem 17](#), parent $(N_m(u), T_m^k) = u$. As a result, $\deg(u, T_m^k) = \deg(u, T_{m-1}^{0,k}) + 1 \geq \min\{k + 2, m\} \geq k + 1$. Therefore, we have $u \in N_m(\text{ST}_{m-1}^k)$. Let $p = \text{parent}(u, T_m^k)$. By the proof of [Theorem 17](#), $p \in \text{ST}_{m-1}^k$. According to above discussion, $u = N_m(p)$.

In summary, the lemma holds for $n = m$. \square

In the following, in order to prove that the n independent spanning trees constructed in the proof of [Theorem 17](#) are isomorphic, we present the definition of an n -dimensional partition tree (PT_n) and some associated lemmas.

Definition 19. For any integer n with $n \geq 1$, an n -dimensional partition tree PT_n rooted at v can be recursively defined as follows.

(1) PT_1 rooted at v is a tree with two vertices, where v has a single child.

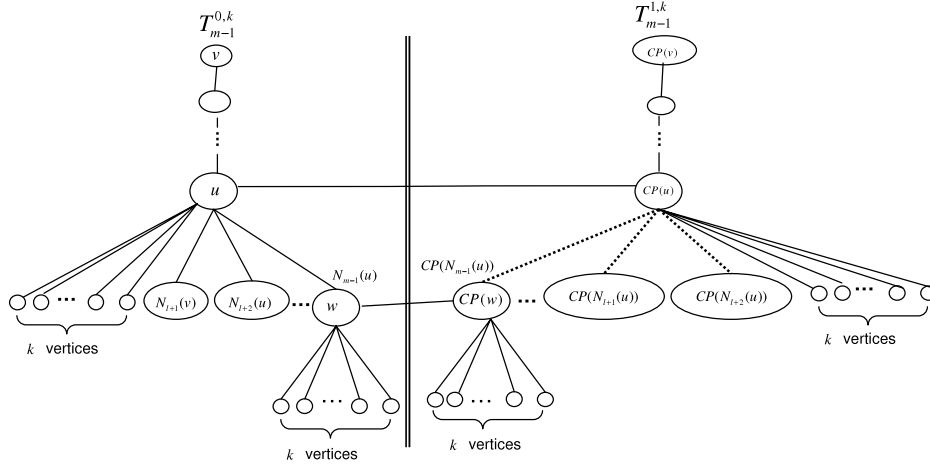


Fig. 5. The computation of $\text{deg}(CP(u), T_m^k)$ where $u \in ST_{m-2}^k$, u is not the single child of root v and $\text{parent}(CP(u), T_m^k) = u$.

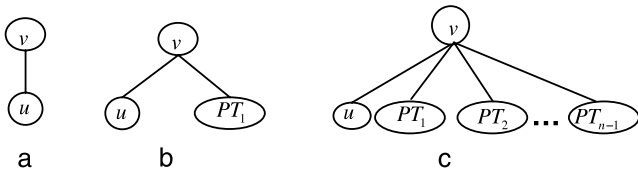


Fig. 6. (a) The graph of PT_1 . (b) The graph of PT_2 . (c) The graph of PT_n .

(2) For any integer $n \geq 2$, an n -dimensional partition tree rooted at v is composed of n subtrees PT_0, PT_1, PT_2, \dots , and PT_{n-1} , where PT_0 is the tree with single vertex and PT_i ($i = 1, 2, \dots, n - 1$) is an i -dimensional partition tree rooted at v_i (see Fig. 6).

By Definition 19, we can easily verify the following lemmas.

Lemma 20. $\text{deg}(v, PT_n) = n$.

Lemma 21. By adding a vertex to each vertex in PT_{n-1} as its child, we can obtain PT_n .

Notation 22. For any integers n and i with $n \geq 3$ and $1 \leq i \leq n - 1$, we use SPT_n^i to denote $T_n^i[ST_n^i]$.

Lemma 23. For any integers n and i with $n \geq 3$ and $1 \leq i \leq n - 1$, $SPT_n^i \cong PT_{n-i}$.

Proof. We will prove the lemma by induction on n . We can easily verify that the lemma holds for $n = 3$ (see Fig. 7(a) and (b)). Supposing that the lemma holds for $n = k - 1$ ($k \geq 4$), we consider $n = k$. By Definition 13, $ST_k^i = ST_{k-1}^i \cup N_k(ST_{k-1}^i)$. For any vertex $u \in ST_{k-1}^i$, by Notation 22, $u \in V(SPT_{k-1}^i)$. By the induction hypothesis, $SPT_{k-1}^i \cong PT_{k-1-i}$ (see Fig. 7(c)). Meanwhile, by the proof of Theorem 17, $\text{parent}(N_k(u), SPT_k^i) = u$. By Lemma 21, we can change PT_{k-1-i} into a $(k - i)$ -dimensional partition tree PT_{k-i} . As a result, $SPT_k^i \cong PT_{k-i}$ (see Fig. 7(d)).

According to the above discussion, the lemma holds for $n = k$. \square

In the following, we will prove that the n independent spanning trees on TQ_n constructed in the proof of Theorem 17 are isomorphic. Obviously, for $n = 1, 2$, the n independent spanning trees on TQ_n constructed in the proof of Theorem 17 are isomorphic. In particular, for $n \geq 3$, we have the following theorem.

Theorem 24. For any integers n and i with $n \geq 3$ and $1 \leq i \leq n$, let T_n^i be an independent spanning tree on TQ_n constructed in the proof of Theorem 17. Then T_n^i is isomorphic to the tree rooted at u that has v as a child of u and v has $n - 1$ subtrees PT_1, PT_2, \dots , and PT_{n-1} .

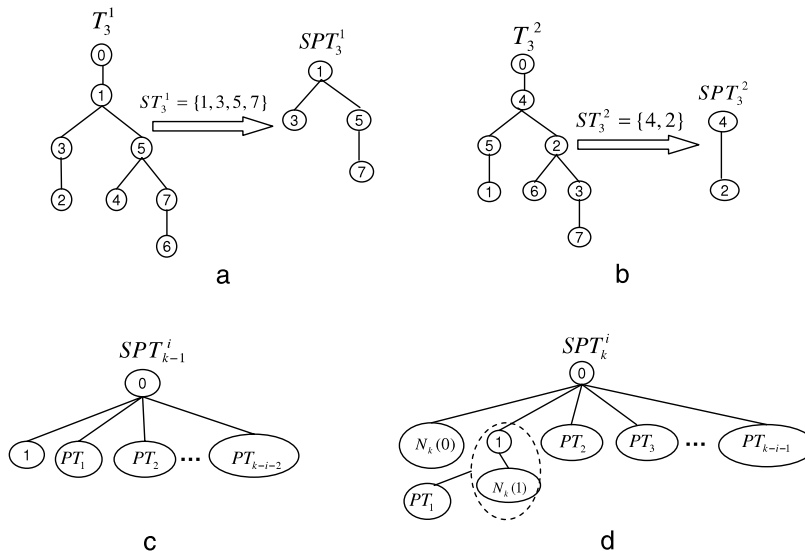


Fig. 7. The illustrations of $SPT_3^1, SPT_3^2, SPT_{k-1}^i$ and SPT_k^i .

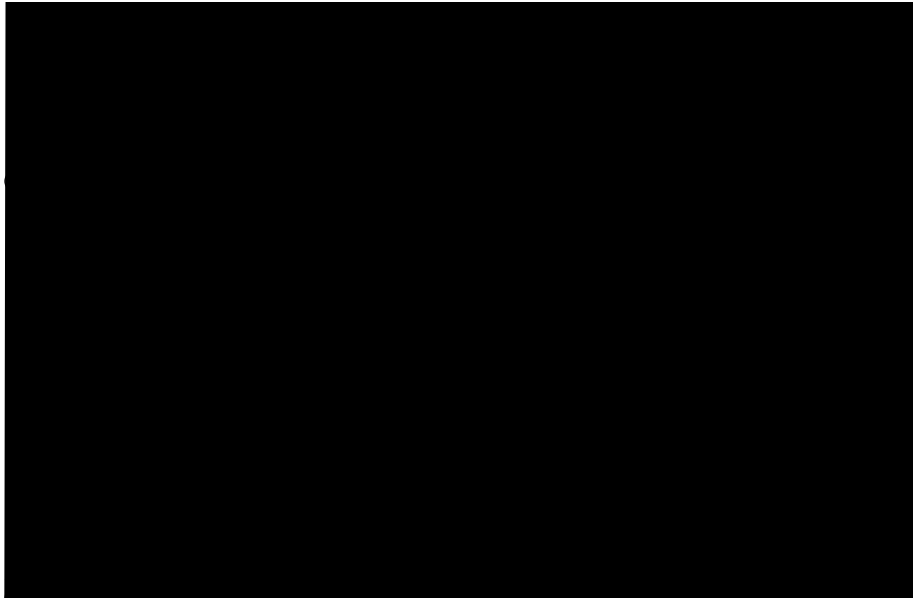


Fig. 8. The construction of T_k^i where $1 \leq i \leq k - 1$, $m = i$ and $N_k(v) = CP(v)$.

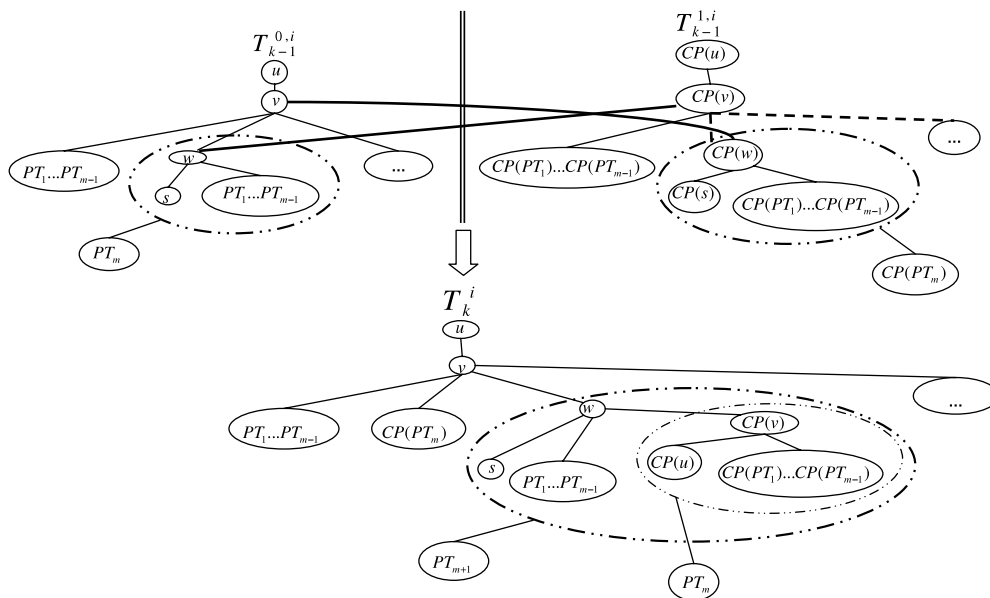


Fig. 9. The construction of T_k^i where $1 \leq i \leq k - 1$, $m = i$ and $N_k(v) = CP(w)$.

Proof. We will prove the theorem by induction on n .

The theorem clearly holds for $n = 3$. Supposing that the theorem holds for $n = k - 1$ ($k \geq 4$) (see Fig. 10(a)), we consider $n = k$. We can deal with the following cases.

Case 1. $1 \leq i \leq k - 1$. For any integer m with $1 \leq m \leq k - 2$, let $w \in V(T_{k-1}^{0,i})$ and $w \neq v$ denote the root of PT_m . By Lemma 20, $\deg(w, PT_m) = m$. Therefore, $\deg(w, T_{k-1}^{0,i}) = m + 1$. We further have the following sub-cases.

Case 1.1. $m < i$. Since $\deg(w, T_{k-1}^{0,i}) < i + 1$, according to the proof of Theorem 17 and Lemma 18, there is no partition vertex in PT_m . Thus PT_m remains unchanged in T_k^i .

Case 1.2. $m = i$. Since $\deg(w, T_{k-1}^{0,i}) = i + 1$, by Lemma 18, w is a partition vertex and $w = N_{k-1}(v)$. By Lemma 14, $N_k(v) = CP(v)$ or $N_k(v) = CP(w)$. Hence, we further have the following sub-cases.

Case 1.2.1. $N_k(v) = CP(v)$. By the proof of Theorem 17, parent $(CP(v), T_k^i) = v$ and parent $(CP(w), T_k^i) = w$. Furthermore, by the proof of Theorem 17, in T_k^i , w will add $CP(PT_m)$ rooted at $CP(w)$ as its subtree. By Lemma 12, $CP(PT_m) \cong PT_m$. And by Definition 19, PT_m in $T_{k-1}^{0,i}$ is changed into PT_{m+1} in T_k^i . Meanwhile, by the proof of Theorem 17, in T_k^i , $CP(PT_1), CP(PT_2), \dots$, and $CP(PT_{m-1})$ in $T_{k-1}^{1,i}$ are changed into $m - 1$ subtrees of $CP(v)$ and the root $CP(u)$ in $T_{k-1}^{1,i}$ is changed into a child vertex of $CP(v)$. By Lemma 12, for any integer l with $1 \leq l \leq m - 1$, $CP(PT_l) \cong PT_l$. Thus in T_k^i , the vertex v will add PT_m rooted at $CP(v)$ as its subtree (see Fig. 8).

Case 1.2.2. $N_k(v) = CP(w)$. By the proof of Theorem 17, parent $(CP(v), T_k^i) = w$ and parent $(CP(w), T_k^i) = v$. By Lemma 12, $CP(PT_m) \cong PT_m$. Thus, v will add PT_m rooted at $CP(w)$ as its subtree. Meanwhile, by the proof of Theorem 17, in T_k^i , $CP(PT_1), CP(PT_2), \dots$, and $CP(PT_{m-1})$ in $T_{k-1}^{1,i}$ are changed into $m - 1$

subtrees of $CP(v)$ and the root in $T_{k-1}^{1,i}$ is changed into a child vertex of $CP(v)$. By Lemma 12, for any integer l with $1 \leq l \leq m-1$, $CP(PT_l) \cong PT_l$. Thus in T_k^i , the vertex w will add PT_m rooted at $CP(v)$ as its subtree. By Definition 19, PT_m in $T_{k-1}^{0,i}$ is changed into PT_{m+1} in T_k^i (see Fig. 9).

Case 1.3. $m > i$. By Lemma 18, there are at least two partition vertices in PT_m . According to Definition 19, Cases 1.1 and 1.2, in T_k^i , the vertex v will add PT_{m+1} as its subtree.

From Cases 1.1–1.3, we can verify the result that for $1 \leq i \leq k-1$, T_k^i is isomorphic to the tree rooted at u that has v as a child vertex of u and v has $k-1$ subtrees PT_1, PT_2, \dots , and PT_{k-1} (see Fig. 10(b)).

Case 2. $i = k$. For any integer j with $1 \leq j \leq k-1$, let s denote any vertex in $V(T_k^k[N_k(ST_{k-1}^j)])$. By the proof of Theorem 17, $\text{parent}(s, T_k^k) = \text{parent}(s, T_{k-1}^{1,j})$. Meanwhile, by Lemma 10, $T_{k-1}^{0,j} \cong T_{k-1}^{1,j}$. Therefore, $T_k^k[N_k(ST_{k-1}^j)] \cong SPT_{k-1}^j$. Then by Lemma 23, $T_k^k[N_k(ST_{k-1}^j)] \cong SPT_{k-1}^j \cong PT_{k-1-j}$. And according to the proof of Theorem 17, for each vertex in $T_k^k[N_k(ST_{k-1}^j)]$, by adding a vertex as its child, we can obtain $T_k^k[N_k(ST_{k-1}^j) \cup ST_{k-1}^j]$. By Lemma 21, $T_k^k[N_k(ST_{k-1}^j) \cup ST_{k-1}^j]$ is a $(k-j)$ -dimensional partition tree PT_{k-j} . As a sequence, T_k^k is isomorphic to the tree rooted at u that has v as a child of u and v has $k-1$ subtrees PT_1, PT_2, \dots , and PT_{k-1} (see Fig. 10(b)).

So far, we have proven that the theorem also holds for $n = k$. Hence, the theorem holds. \square

4.2. Height of independent spanning trees

In order to analyze the height of independent spanning trees, we first give the following lemma.

Lemma 25. For any integer n with $n \geq 1$, $\text{Height}(PT_n) = n$.

Proof. We will prove the lemma by induction on n . The lemma clearly holds for $n = 1$. Supposing that the lemma holds for $n = k-1$ ($k \geq 2$), we consider $n = k$. By Definition 19, $\text{Height}(PT_k) = \text{Height}(PT_{k-1}) + 1$. By the induction hypothesis, $\text{Height}(PT_{k-1}) = k-1$. Therefore, $\text{Height}(PT_k) = k$. \square

Theorem 26. For any integer n with $n \geq 2$, the height of each independent spanning tree on TQ_n constructed by the proof of Theorem 17 is $n+1$.

Proof. For $n = 2$, it is obvious that the theorem holds. For $n \geq 3$, by Theorem 24, all independent spanning trees are isomorphic to the graph for $n = k$ shown in Fig. 10(b). Furthermore, by Lemma 25, $\text{Height}(PT_{n-1}) = n-1$. Thus the height of each independent spanning tree on TQ_n constructed by the proof of Theorem 17 is $\text{Height}(PT_{n-1}) + 2 = n+1$. \square

5. Algorithm TQIST and its analysis

Based on the proof of Theorem 17, we provide an algorithm to generate n independent spanning trees rooted at any vertex v on TQ_n . The algorithm can be described as follows.

5.1. Algorithm TQIST

5.2. Time complexity of Algorithm TQIST

Based on Algorithm TQIST, in the following, we will analyze the time complexity of Algorithm TQIST.

Theorem 27. Algorithm TQIST constructs n independent spanning trees on TQ_n in $O(N \log N)$ time, where $N = 2^n$ is the number of vertices in TQ_n .

Proof. Based on Algorithm TQIST, let $f(N)$ denote the running time of Algorithm TQIST. Then we have a recurrence equation that

Algorithm 1 TQIST

Input: A positive integer n and $v \in V(TQ_n)$

Output: A set of n independent spanning trees rooted at vertex v .

BEGIN

if $n = 1$ **then**

$T_1^1 = (\{v, N_1(v)\}, \{(v, N_1(v))\}); ST_1^1 = \{N_1(v)\};$

else

call TQIST($n-1, v$);

end if

$ST_n^n = N_n(v);$

$\text{parent}(N_n(v), T_n^n) = v;$

for $1 \leq i \leq n-1$ **do**

construct $T_{n-1}^{1,i}$ according to Rule 9; $ST_n^i = ST_{n-1}^i \cup N_n(ST_{n-1}^i);$

$V(T_n^i) = V(T_{n-1}^{0,i}) \cup V(T_{n-1}^{1,i}); E' = \{(u, N_n(u)) | u \in ST_{n-1}^i\};$

if $v \in V(TQ_{n-1}^0)$ **then**

for each vertex $u \in ST_{n-1}^i$ **do**

$\text{parent}(N_n(u), T_n^i) = u;$

end for

$E'' = E(T_{n-1}^{1,i}[N_n(ST_{n-1}^i)]); E(T_n^i) = (E(T_{n-1}^{0,i}) \cup E(T_{n-1}^{1,i}) \cup E') - E'';$

for each vertex $u \in N_n(ST_{n-1}^i)$ **do**

$\text{parent}(u, T_n^i) = \text{parent}(u, T_{n-1}^{1,i});$

end for

for each vertex $w \in N_n(ST_{n-1}^i)$ **do**

$\text{parent}(w, T_n^i) = N_n(w);$

end for

else

for each vertex $u \in ST_{n-1}^i$ **do**

$\text{parent}(u, T_n^i) = N_n(u)$

end for

$E'' = E(T_{n-1}^{0,i}[ST_{n-1}^i]); E(T_n^i) = (E(T_{n-1}^{0,i}) \cup E(T_{n-1}^{1,i}) \cup E') - E'';$

for each vertex $u \in ST_{n-1}^i$ **do**

$\text{parent}(u, T_n^i) = \text{parent}(u, T_{n-1}^{0,i});$

end for

for each vertex $w \in N_n(ST_{n-1}^i)$ **do**

$\text{parent}(w, T_n^i) = N_n(w);$

end for

end if

end for

END

bounds $f(N) : f(N) = 2f(N/2) + cN$, where c is a constant. Solve this recurrence equation, then we can obtain $f(N) = O(N \log N)$. \square

6. Conclusions

There are two versions for the n independent spanning trees conjecture. The vertex (edge) conjecture is that any n -connected (n -edge-connected) graph has n vertex-independent (edge-independent) spanning trees rooted at an arbitrary vertex. It has been proven that the vertex conjecture implies the edge conjecture. In this paper, we propose a construction method to construct n isomorphic vertex-independent spanning trees rooted at any vertex on the n -dimensional twisted cube and the height of each independent spanning tree is $n+1$ for any integer n with $n \geq 2$. In addition, we provide an $O(N \log N)$ algorithm to construct n independent spanning trees rooted at any vertex on TQ_n , where N denotes the number of vertices in TQ_n .

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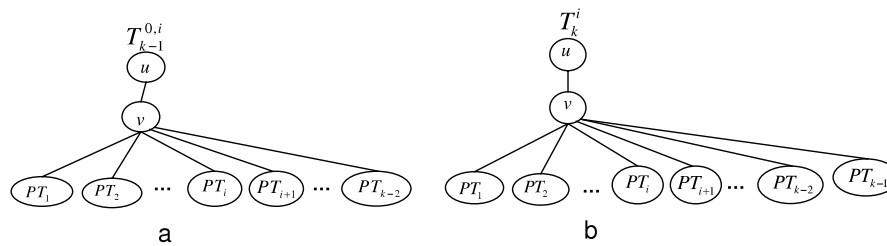


Fig. 10. (a) An illustration of $T_{k-1}^{0,i}$. (b) An illustration of T_k^i .

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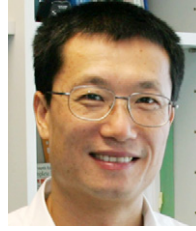


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